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STUDENT ATTAINMENT IN ALGEBRA 2
EXAMINATION OF 1944

AN ANALYSIS, AND COMPARISON, OF THE ERRORS MADE, ON THE 1944 DEPARTMENTAL EXAMINATION PAPER IN ALGEBRA 2, BY THE ONE HUNDRED STUDENTS WHOSE MARKS WERE IMMEDIATELY ABOVE 39% AND BY THE ONE HUNDRED STUDENTS WHOSE MARKS WERE IMMEDIATELY ABOVE 49%

A DISSERTATION

SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR

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BY

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CHAPTER I

PURPOSE OF THE INVESTIGATION

Foreword

From the Examinations Branch of the Department of Education were secured the 1944 Algebra 2 final examination booklets for all students whose raw scores were 42, 43, 44, 45, 46 and 47 and for as many students whose raw score was 48 as were required to bring the total number of papers up to one hundred. These raw scores corresponded to percentages 40 to 42. Thus, these papers were the work of the one hundred students who received percentages immediately above 39. Also were secured the booklets for all students whose raw scores were 65, 66, 67, 68 and 69 and for as many students whose raw score was 70 as were required to bring the total number of papers up to one hundred. These raw scores corresponded to percentages 50 to 53. Thus, these papers were the work of the one hundred students who received percentages immediately above 49.

A copy of the scoring key used by the

original markers was of value as a guide in carrying out this investigation.

Throughout this report Group B will signify that group of students who made percentages immediately above 49, and Group C that group who made percentages immediately above 39. Tables I and II below indicate in more detail the composition of Groups B and C.

TABLE I

DISTRIBUTION OF SCORES AND PERCENTAGES IN GROUP C

Score	Percentage	Number of Papers
42	40	9
43	40	21
44	41	19
45	41	14
46	41	11
47	42	19
48	42	15

TABLE II

DISTRIBUTION OF SCORES AND PERCENTAGES IN GROUP B

Score	Percentage	Number of Papers
65	50	12
66	51	23
67	51	19
68	52	13
69	52	19
70	53	14

Purpose of the Investigation

The purpose of the investigation, then, was to compare and contrast the work of students in Groups B and C. By analyzing the errors made by all an attempt was made to discover why a given student fell in Group C and not in Group B --that is, to discover whether or not this division into groups was a fair one. In other words it was desired to determine whether or not the difference in attainment between the two groups of students was significant. An attempt was also made to answer such questions as : Was the Algebra 2 exam of 1944 a valid test? Which

items were good test items? Which were poor ones?

The sub-title, then, indicates clearly the purpose of the investigation, and reads as follows: "An analysis, and comparison, of the errors made, on the 1944 Departmental Examination in Algebra 2, by the one hundred students whose marks were immediately above 39% and by the one hundred students whose marks were immediately above 49%".

a given reading was fair or correct. The many variables enter into the problem--the emotional state of the individual, the reading set, the cultural background, the mental capacity, the kind of teaching received, etc. It is questionable whether or not the tests in this study were adequate, even the generic coefficients for fitness and validity of the reading tests or inventory.

CHAPTER II

COMPARISON OF SCORES ON INDIVIDUAL TEST ITEMS

Frequencies[1] of Scores on the Various Items

The following frequency tables and graphs indicate how many students received a given mark for a certain question. For items 1 to 25 a perfect score was 2 and the questions were marked either right or wrong (i.e. scored 2 or 0), except questions 4, 6, 9, 14, 15, 23 and 25 for which partial answers were accepted and scored 1. For items 26 to 44 perfect scores varied from 4 to 7 and partial answers were accepted in all cases.

TABLE III

FREQUENCIES OF SCORES ON ITEMS 1 TO 5

Question	1		2		3		4		5	
Group / Score	B	C	B	C	B	C	B	C	B	C
0	25	22	9	15	29	45	16	24	61	77
1	0	0	0	0	0	0	7	16	0	0
2	75	78	91	85	71	55	77	60	39	23

[1] In this chapter frequencies are identical with percentages because the total number of cases in each group is 100.

TABLE IV

FREQUENCIES OF SCORES ON ITEMS 6 TO 10

Question	6		7		8		9		10	
Group	B	C	B	C	B	C	B	C	B	C
Score										
0	27	40	24	43	12	17	67	88	43	49
1	6	15	0	0	0	0	11	7	0	0
2	67	45	76	57	88	83	22	5	57	51

TABLE V

FREQUENCIES OF SCORES ON ITEMS 11 TO 15

Question	11		12		13		14		15	
Group	B	C	B	C	B	C	B	C	B	C
Score										
0	69	82	3	9	41	48	74	92	28	63
1	0	0	0	0	0	0	2	0	40	30
2	31	18	97	91	59	52	24	8	32	7

TABLE VI

FREQUENCIES OF SCORES ON ITEMS 16 TO 20

Question	16		17		18		19		20	
Group	B	C	B	C	B	C	B	C	B	C
Score										
0	28	43	43	71	37	64	45	59	42	61
1	0	0	0	0	0	0	0	0	0	0
2	72	57	57	29	63	36	55	41	58	39

TABLE VII

FREQUENCIES OF SCORES ON ITEMS 21 TO 25

Question	21		22		23		24		25	
Group / Score	B	C	B	C	B	C	B	C	B	C
0	13	22	41	46	57	58	22	50	9	16
1	0	0	0	0	8	15	0	0	11	11
2	87	78	59	54	35	27	78	50	80	73

TABLE VIII

FREQUENCIES OF SCORES ON ITEM 26

Group / Score	B	C
0	42	54
1	1	2
2	1	4
3	1	0
4	55	40

TABLE IX

FREQUENCIES OF SCORES ON ITEM 27

Group / Score	B	C
0	62	76
1	7	4
2	20	16
3	0	1
4	0	1
5	0	0
6	11	2

TABLE I

FREQUENCIES OF SCORES ON ITEM 28

Score	Group B	C
0	8	24
1	1	3
2	15	14
3	4	1
4	8	7
5	6	8
6	58	43

TABLE II

FREQUENCIES OF SCORES ON ITEM 29

Score	Group B	C
0	14	21
1	14	26
2	15	20
3	11	8
4	15	8
5	9	5
6	21	8

TABLE XII

FREQUENCIES OF SCORES ON ITEM 30

Score	Group B	C
0	60	78
1	12	7
2	6	6
3	7	7
4	13	4

TABLE XIII

FREQUENCIES OF SCORES ON ITEM 31

Score	Group B	C
0	39	71
1	14	19
2	4	3
3	5	2
4	6	3
5	4	2
6	8	0

TABLE XIV

FREQUENCIES OF SCORES ON ITEM 32

Score	Group E	C
0	50	60
1	0	0
2	10	11
3	1	1
4	25	1
5	0	0
6	5	1

TABLE XV

FREQUENCIES OF SCORES ON ITEM 33

Score	Group E	C
0	64	71
1	2	1
2	1	13
3	1	2
4	4	1
5	14	3

TABLE XVI

FREQUENCY DIST. OF SCORES ON [illegible]

Score	Group [illegible]	C
0	12	41
1	3	3
2	2	4
3	3	2
4	2	1
5	31	11

TABLE XVII

FREQUENCIES OF SCORES ON [illegible] 35

Score	Group B	C
0	20	36
1	25	23
2	8	12
3	11	3
4	7	10
5	6	3
6	24	9

TABLE XVIII

FREQUENCIES OF SCORES OF THE JO

Score	Group B	Group C
0	0	7
1	16	4
2	8	1
3	0	0
4	4	1
5	4	3

TABLE XIX

FREQUENCIES OF SCORES OF THE JW

Score	Group B	Group J
0	10	8
1	12	10
2	3	3
3	17	12
4	13	8
5	11	12
6	14	7

Table [illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Table [illegible]

[illegible]

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

TABLE XXII

[illegible]

[illegible]	[illegible]	
0	46	0
1	3	1
2	2	2
3	1	3
4	0	4
5	1	5
6	1	6

TABLE XXIII

[illegible]

[illegible]	[illegible]	
0	0	0
1	1	1
2	1	2
3	1	3
4	1	4
5	0	5

TABLE XXIV

FREQUENCIES OF SCORES IN TEST A

Score	B	C
7	0	14
6	2	8
5	4	5
4	11	3
3	13	2
2	20	0
1	25	0

TABLE XXV

FREQUENCIES OF SCORES IN TEST B

Score	B	C
8	30	7
7	23	3
6	7	1
5	3	1
4	2	1
3	1	1

TABLE XXV

FREQUENCIES OF SCORES OF [illegible]

Score	General	[illegible]	J
0	[illegible]	[illegible]	42
1	[illegible]	[illegible]	7
2	[illegible]	[illegible]	2
3	[illegible]	[illegible]	14
4	[illegible]	[illegible]	1
5	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	0

Some diagrams will illustrate the general trends. Of the graphs which follow, figures 1 and 6 are most representative--that is, these two types of distributions were found most frequently. A total of five distributions similar to that pictured in figure 1, and a total of four distributions similar to that pictured in figure 6, were found. Distributions of the types depicted in figures 2, 3 and 4 were each found to occur twice. Each of the remaining four distributions had its own distinctive characteristics. The relative merits of the individual questions will be dealt with in a later chapter.

Frequency Table

SCORE	0	1	2	3	4
Group B	42	1	1	1	55
Group C	54	2	4	0	40

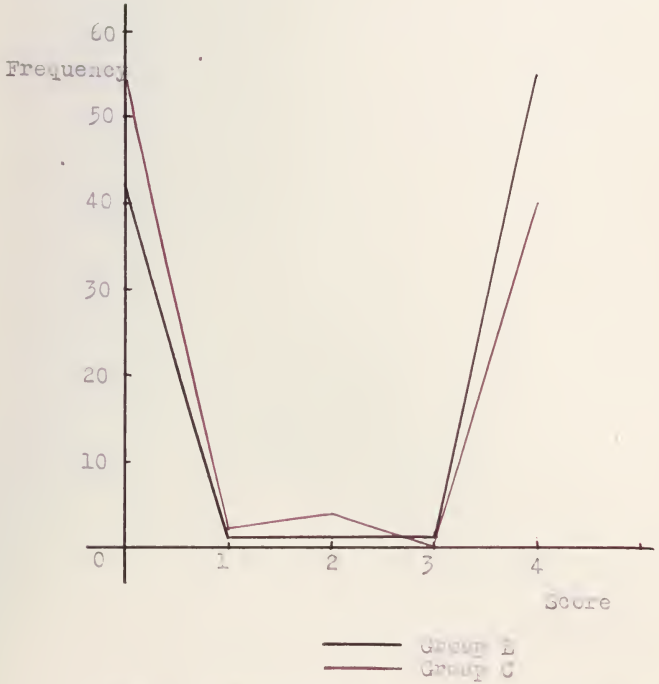

Fig. 1.- Frequencies of scores on inversion 25.

Frequency Table

Scores	0	1	2	3	4	5	6
Group B	8	1	15	4	8	6	58
Group C	24	3	14	1	7	8	43

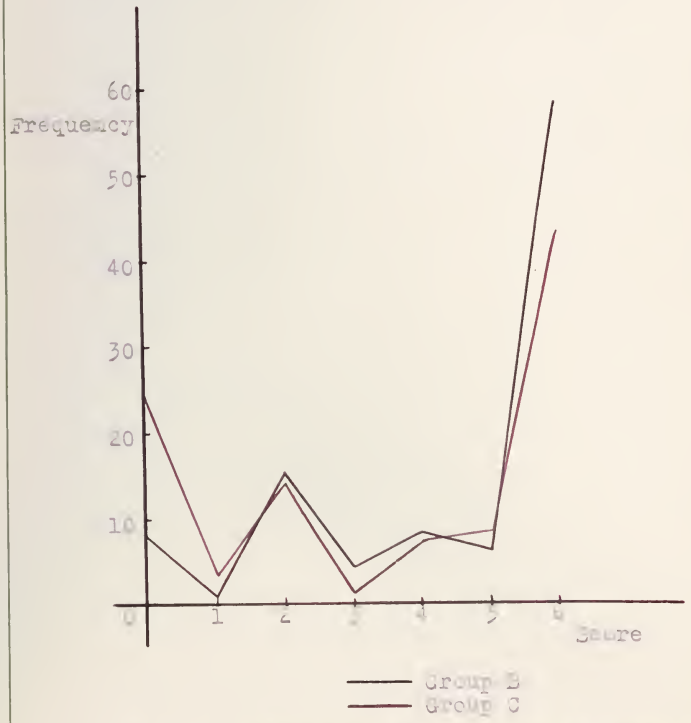

Fig. 2. - Frequencies of scores on question 28.

Frequency Table

SCORE	0	1	2	3	4	5	6
Group E	14	14	15	11	15	9	22
Group C	21	26	26	8	6	5	8

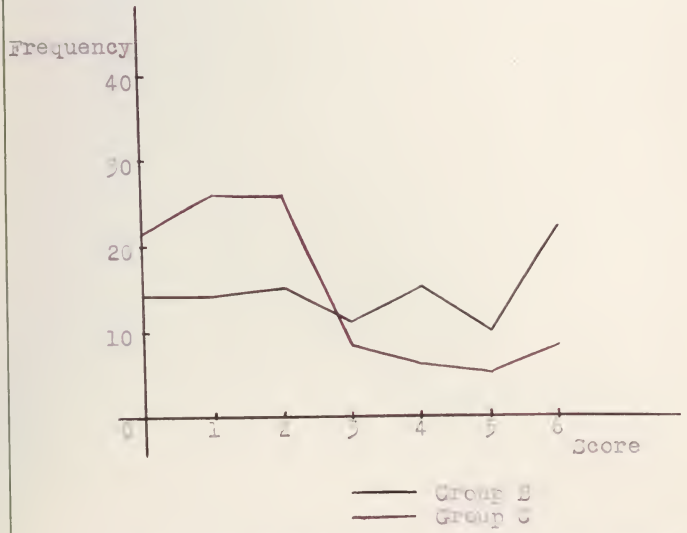

Fig. 3. - Frequencies of scores on question 29.

Frequency Table

SCORE	0	1	2	3	4	5
Group B	68	16	8	0	4	4
Group C	87	4	8	0	1	0

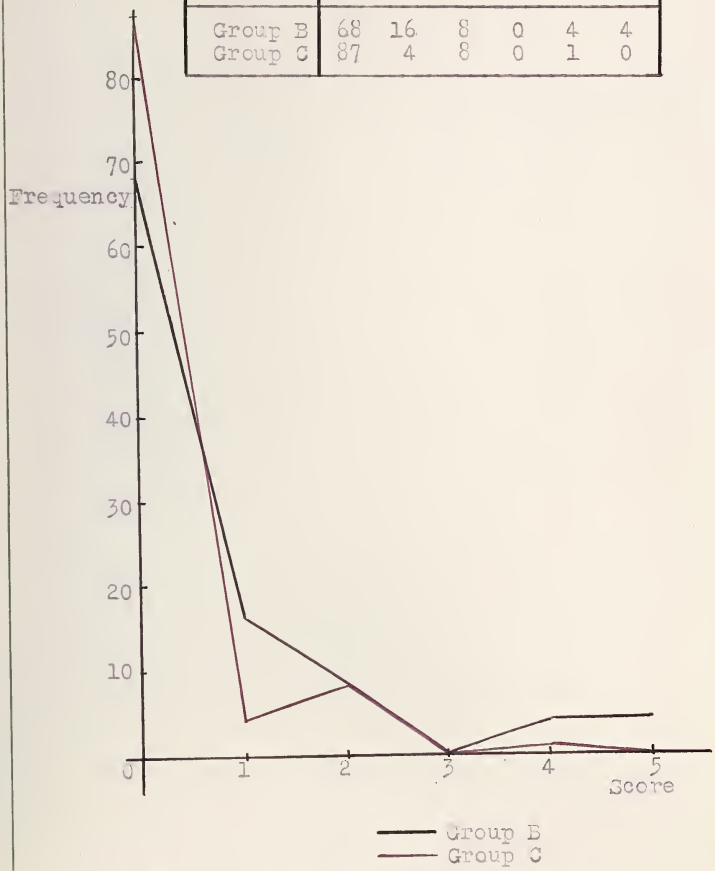

Fig. 4. - Frequencies of scores on question 36.

Frequency Table

SCORE:	0	1	2	3	4	5	6
Group B	10	12	3	17	13	21	14
Group C	48	16	7	10	1	11	7

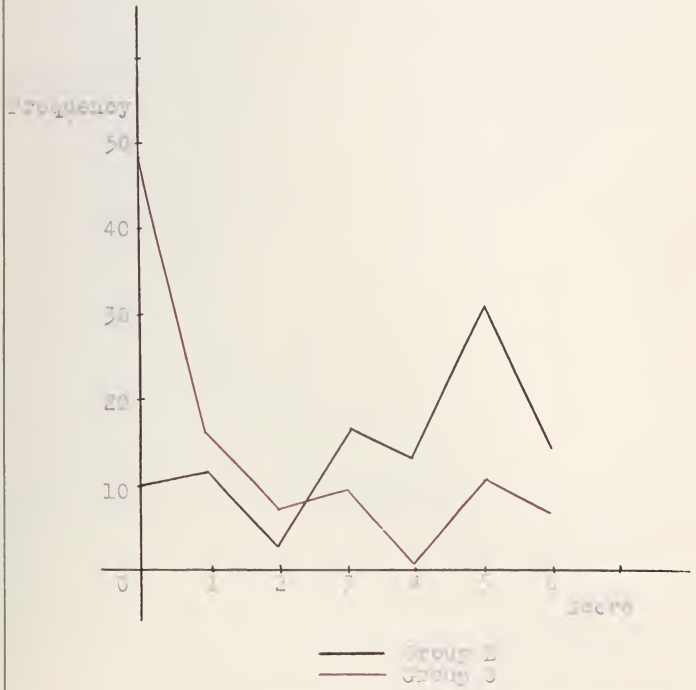

Fig. 3.- Frequencies of scores on question 37.

Frequency Table

SCORE	0	1	2	3	4	5
Group B	47	21	6	9	8	9
Group C	78	9	7	5	0	1

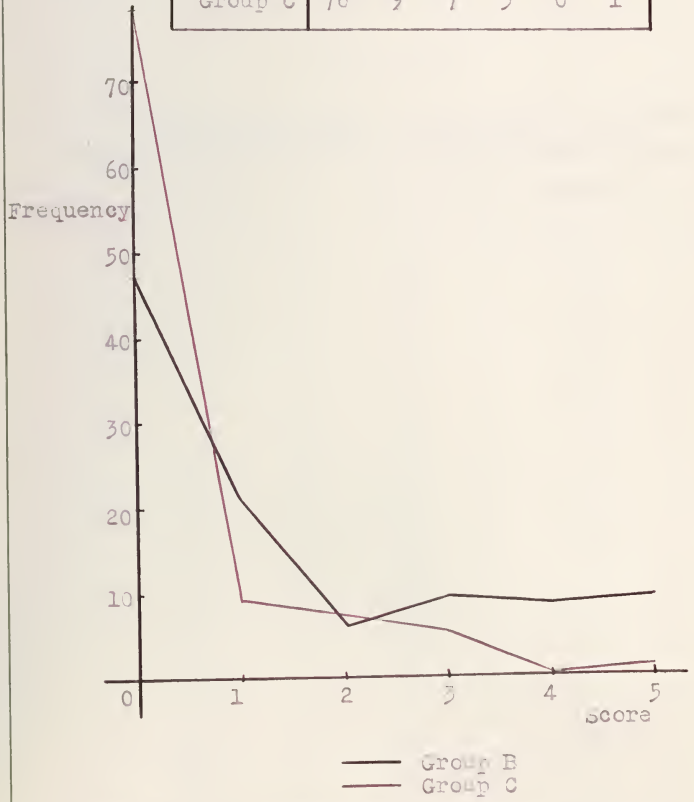

Fig. 6. - Frequencies of scores on question 43.

Frequencies of Perfect Scores

The tables and graphs immediately below tell the story quite clearly.

TABLE XXVII

COMPARISON OF FREQUENCIES OF PERFECT[a] SCORES IN GROUPS B AND C, QUESTIONS 1 TO 25

Question	Group B	Group C
1	75	78
2	91	85
3	71	55
4	77	60
5	39	23
6	67	45
7	76	57
8	88	83
9	22	5
10	57	51
11	31	18
12	97	91
13	59	52
14	24	8
15	32	7
16	72	57
17	57	29
18	63	36
19	55	41
20	58	39
21	87	78
22	59	54
23	35	27
24	78	50
25	80	73

[a] Perfect score is 2.

TABLE XXVIII

COMPARISON OF FREQUENCIES OF PERFECT[b] SCORES IN GROUPS B AND C, QUESTIONS 26 TO 44

Question	Group B	Group C
26	55	40
27	11	2
28	58	43
29	22	8
30	13	4
31	8	0
32	5	1
33	14	3
34	53	22
35	28	9
36	4	0
37	14	7
38	0	0
39	3	1
40	9	3
41	6	5
42	23	12
43	9	1
44	0	0

[b] Perfect score varies from 4 to 7.

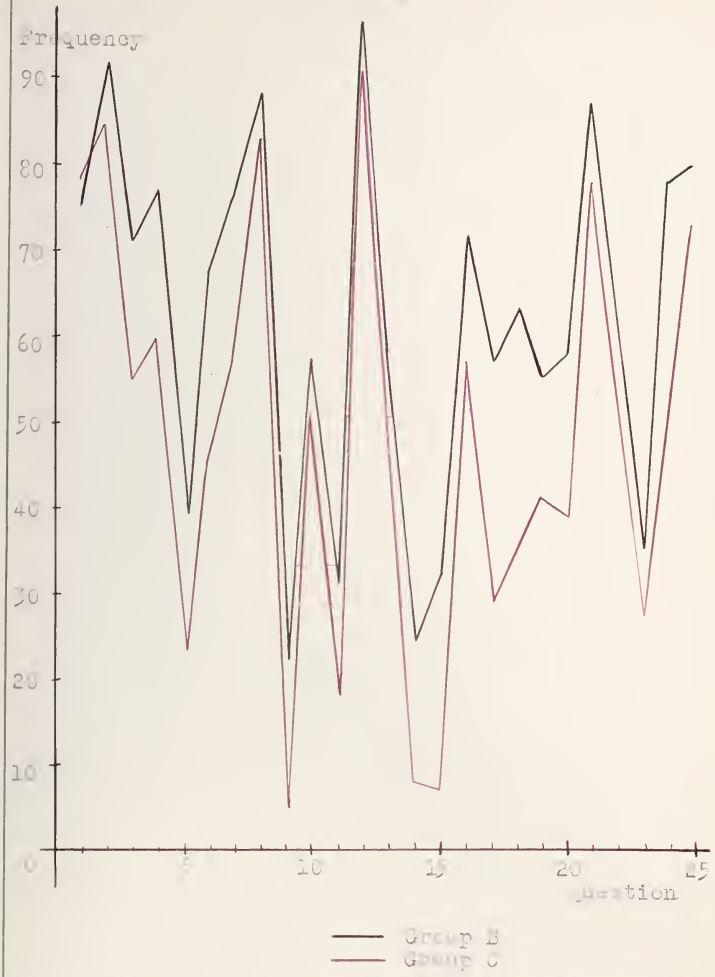

Fig. 7. - Frequencies of perfect scores on questions 1 to 25.

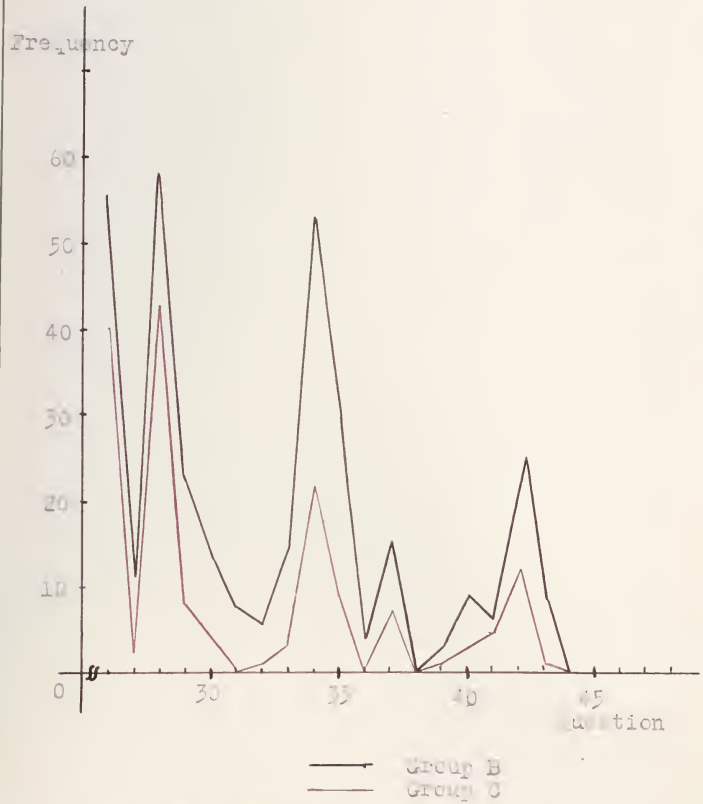

Fig. 8. - Frequencies of perfect scores on questions 26 to 44.

Conclusion

Students in group B were more successful than those in group C on all items with but one exception--question 1--on which 78 group C students obtained perfect scores as compared with 75 group B students. In group B the percentages who obtained perfect scores varied from 0 to 97. In group C these percentages varied from 0 to 91.

CHAPTER III

ANALYSIS OF ERRORS

Mechanical Errors

Many mechanical errors, of many different types, were found. These are tabulated below. Most of the errors were in the handling of simple numbers, fractions and algebraic expressions. Some examples are: (1) $291 - 191 = 190$, (2) $6x7x6 = 230$, (3) $\sqrt{41} = 6.041$, (4) $32/5 = 6.04$, (5) $(x^2)^{10} = x^{12}$, (6) $(t + 1/6)^2 = t^2 + 1/t^2$, (7) $\sqrt{25 - x^2} = 5 - x$, and (8) 3 substituted in formula instead of 3^2.

TABLE XXIX

FREQUENCIES OF NUMERICAL ERRORS

Type	Group B	Group C
Arithmetic of small numbers:		
Addition and subtraction	4	3
Multiplication and division	13	11
Extraction of square roots	6	7
Fractions:		
Addition and subtraction	21	6
Multiplication and division	19	16
Decimals:		
Addition and subtraction	4	1
Multiplication and division	10	15
Relation between decimals and fractions	0	3
Indices:		
Multiplication and division	67	72
Ratio:		
Inverted	2	1
Algebra:		
Squaring	13	11
Extraction of square roots	2	1
Multiplication and division	6	2
Solution of equations	4	2
Substitution	16	9
Addition of fractions	1	5
Logarithms:	2	4
Copying errors:	13	3
Miscellaneous:	3	4
TOTALS:	206	178

In some cases it was possible to ascertain what percent of the students who attempted a certain operation made a mechanical error. The results of this survey are shown in Table XXX.

TABLE XXX

COMPARISON OF NUMBERS OF MECHANICAL ERRORS WITH NUMBERS OF ATTEMPTS

Question and Operation		Errors	Attempts	%
9. Extraction of square root of 41:	B--	6	51	11.8
	C--	7	28	25.0
23. $(2x^3)^5$: Multiplication of indices:	B--	43	77	55.8
	C--	42	69	60.9
35. $(t + 1/t)^2$: Squaring an algebraic expression:	B--	12	47	25.5
	C--	11	23	47.8
43. $(x^2)^{10}$: Multiplication of indices:	B--	24	59	40.7
	C--	30	56	53.6

These figures give a truer picture of the situation. In each case, when the number of errors made is compared with the number of attempts, members of group B are found to be more accurate than members of group C. In other words, although group B was found to have made more errors than group C, this may be explained away by the fact that group B attempted a larger number of operations.

It may be concluded from these data, then, that the students in group B were superior to students in group C insofar as mechanical operations were concerned.

Statistical Analysis of Errors Made in the Various Items

On the following pages is given each of the examination questions together with an analysis of the errors made by the students in their attempts to answer the question.

Question 1.--What fraction with denominator 33 is equivalent to $\frac{1\ 2/3}{11}$?[a]

TABLE XXXI

ANALYSIS OF ERRORS MADE IN QUESTION 1

Explanation	Group B[b]	Group C[b]
Correct answer: 5/33	75	78
Gave numerator 5 only (language difficulty)	14	9
Confused attempts: e.g. 5533	9	10
No attempt	2	3

[a] In questions 1 to 25, answers only were required.
[b] For explanation of errors.

Question 1, which dealt with fractions, was answered equally successfully by the two groups.

Question 2.--If 5, a, 10, 3 are in proportion, find a.

TABLE XXXII

ANALYSIS OF ERRORS MADE IN QUESTION 2

Explanation	Group B	Group C
Correct answer: a = 1.5 (from equation 10a = 15)	91	85
a = 6 (5a = 30)	1	1
a = 450 (5/a = a/10)	1	2
a = 16 2/3 (3a = 50, a/5 = 5/10)	1	1
Mechanical errors	1	0
Other confused attempts	5	6
No attempt	0	5

Members of group B were successful in receiving a perfect score for question 2, which concerns proportion, in 91% of the cases as compared with 85% for group C. Both groups made about the same numbers and types of errors. The difference in percentage of success was caused by no attempt on the part of five students in group C.

Question 3.--U and V are related variables. If doubling U always produces the effect of halving V, what is the variation-relation between U and V?

TABLE XXXIII

ANALYSIS OF ERRORS MADE IN QUESTION 3

Explanation	Group B	Group C
Correct answer: $U \propto 1/V$	71	55
$U^2 \propto \frac{1}{2}V$	3	1
$U \propto V$	2	5
$U \propto \frac{1}{2}V$ or $U = \frac{1}{2}kV$	12	18
$V = 1/U$ or $U/2 \propto K$	2	1
$U \propto 1/2V$ or $V \propto 1/2U$	2	0
$U^2 \propto 1/V$	1	0
$U \propto 1/V^2$ or $U = 1/V^2 \cdot k$	1	2
Used symbol '=' instead of '$\propto$'	11	11
Others	4	16
No attempt	1	1

The solutions given for question 3 (on variation) indicate the definite superiority of the students in group B. However, members of both groups seemed to have considerable difficulty with the language of 'variation' and gave a great variety of answers. Many did not grasp the fundamental concept of variation, but used the sign of equality indiscriminantly instead of the sign of variation.

Question 4.--At what three points does the graph of the function $x^3 - x$ cross the x-axis?

TABLE XXXIV

ANALYSIS OF ERRORS MADE IN QUESTION 4

Explanation	Group B	Group C
Correct answer: -1, 0, +1	77	60
Gave two correct points only	7	16
Gave answers involving x: $-x, x^2$,	2	2
Gave factors: $x(x-1)(x+1)$	1	1
Other incorrect answers: e.g. 2; 1, 6, -6; 0	6	12
No attempt	7	9

Here, also, group B members were more successful.

Question 5.--Express the area (A) of a square as a function of its perimeter (p).

TABLE XXIV

ANALYSIS OF ERRORS MADE IN QUESTION 5

Explanation	Group B	Group G
Correct answer: $A = \frac{p^2}{16}$	39	23
$A = p^2/4$	7	11
$A = r^2$	2	5
$A = p^2/2$	2	0
$A = p/4$	2	2
$p = 4\sqrt{A}$	0	5
$A = 2p$	1	2
$A = s^2$, $p = 4s$ and no further	6	10
Used $A = 1s$, didn't note that $1 = s$	4	3
$A \propto p$	0	3
$A \propto p^2$	0	2
Other confused attempts	28	27
No attempt	8	7

Students in both groups found this item more difficult, yet those in group B again did better work. So many varied answers would indicate a lack of knowledge of the language and concepts of variation and functionality.

Question 6.--The diagram below shows the graph of the function $4x^3 - 16x^2 - 9x + 36$. Use this graph to solve the equation $4x^3 - 16x^2 - 9x + 36 = 0$, giving answers to the nearest tenth.

TABLE XXXVI

ANALYSIS OF ERRORS MADE IN QUESTION 6

Explanation	Group E	Group C
Correct answer: $-1\frac{1}{2}, +1\frac{1}{2}, +4$,	67	45
Gave only 2 correct solutions	6	15
Gave only 1 correct solution(+4)	3	2
Introduced factors	5	7
'3' given as answer	1	1
Other confused attempts	11	13
No attempt	7	17

Students in group E were superior to students in group C in their efforts to answer question 6.

Question 7.--The roots of the quadratic equation $x^2 - 6x + C = 0$ are equal. Find the value of C.

TABLE XXXVII

ANALYSIS OF ERRORS MADE IN QUESTION 7

Explanation	Group B	Group C
Correct answer: C = 9	76	57
C = -3	5	3
C = +3	3	1
C = $6x - x^2$	2	3
C = 6	1	2
C = 0	2	2
Other confused attempts	6	11
No attempt	5	22

Group B again gave evidence of superiority over group C.

Question 8.--If $f(x) = \frac{x^2 + 1}{x}$, find the value of $f(-1)$.

TABLE XXXVIII

ANALYSIS OF ERRORS MADE IN QUESTION 8

Explanation	Group B	Group C
Correct answer: -2	88	83
+2	6	10
0 (error in squaring -1)	2	0
-1 (mechanical error)	2	4
Other confused attempts	2	2
No attempt	0	1

On the whole, members of both groups found this question relatively easy. Most errors made were mechanical errors.

Question 9.--Solve the equation $2x^2 - 3x - 4 = 0$, giving the roots correct to the nearest second decimal place.

TABLE XXXIX

ANALYSIS OF ERRORS MADE IN QUESTION 9

Explanation	Group E	Group C
Correct answer: x = 2.35, -.85	22	5
Did not use correct formula	14	13
Mech. error in evaluating $\sqrt{41}$	6	7
Error in evaluating $b^2 - 4ac$ (33, 25, etc. instead of 41)	11	17
Other errors in evaluating formula	14	7
Tried to factor	7	14
Tried to 'complete the square'	6	4
Reversed the signs: -2.35, +.85	6	3
Gave one correct value only	11	7
Other confused attempts	2	7
No attempt	8	16

Students of both groups had great difficulty with this question. Many did not know how to tackle the problem. Many others did not know the formula. However, once again members of group E showed their superiority.

Question 10.--As n increases without limit, find the limit approached by $\frac{12}{n-10}$.

TABLE XL

ANALYSIS OF ERRORS MADE IN QUESTION 10

Explanation	Group B	Group C
Correct answer: 0	57	31
Language difficulty: said 'decreases without limit', 'something very small', 'limit approaches 0', etc.	8	2
12	1	6
$-\infty$	1	3
$+\infty$	7	6
$\frac{1}{\infty}$; $\frac{12}{\infty}$	3	3
-1.2	5	10
1	3	6
-1	2	1
Other common attempts: 1.2, 1, etc.	9	11
No attempt	7	3

Members of group B were only slightly more successful with this question--that is, only slightly more successful than were members of group C. In approximately 45% of the cases (in both groups) students did not seem to understand the concept of a 'limit'. Many used the symbol '∞' indiscriminantly as if it were a definite number.

Question 11.--Find the gradient of the straight line which has x and y intercepts 3 and -2 respectively.

TABLE XLI

ANALYSIS OF ERRORS MADE IN QUESTION 11

Explanation	Group B	Group C
Correct answer: 2/3	31	18
-2/3	38	46
-3/2	6	8
3/2	1	3
Other incorrect answers: 3, 6, 2, etc.	9	13
No attempt	15	10

A large number of students (38% of group B and 46% of group C) merely found the ratio of the y intercept over the x intercept and thus found a negative, instead of a positive, slope. Once more the members of group B showed their superiority.

uestion 12.--Given $y = 3x^5$. Find dy/dx.

TABLE XLII

ANALYSIS OF ERRORS MADE IN QUESTION 12

Explanation	Group B	Group C
Correct answer: $15x^4$	97	91
$15x^2$	2	3
Other errors: $15x$, $3/5\ x^4$, etc.	5	1
No attempt	0	1

Practically all students of both groups were successful here. Apparently this question was too easy, otherwise students of the calibre dealt with in this report, would not have done so well. The majority of students seemed to know how to differentiate simple expressions. Quite likely most errors were mechanical.

Question 13.--What is the smallest value of the function $x^2 + 5$?

TABLE XLIII

ANALYSIS OF ERRORS MADE IN QUESTION 13

Explanation	Group B	Group C
Correct answer: 5	59	52
0	12	6
$\sqrt{5}$, $-\sqrt{5}$ or $\pm\sqrt{5}$	7	6
-5	0	3
2 or -2	0	6
Other incorrect answers: 1, -5, etc.	11	15
No attempt	11	11

Members of group B were successful in only 7% more cases than were members of group C. A large percentage showed no insight into a rather simple question dealing with 'minimum value' of a simple function.

Question 14.--Given $S = \frac{t^3}{5} + 1$, and assuming that $\frac{dS}{dt} \cdot \Delta t$ is an approximation for ΔS, find the approximate change in S when t increases from 2.9 to 3.1.

TABLE XLIV

ANALYSIS OF ERRORS MADE IN QUESTION 14

Explanation	Group B	Group C
Correct answer: $\Delta S = 1.682$	24	8
Went as far as $.2t^2$ only	8	4
Went as far as $t^2 \cdot \Delta t$ only	3	2
Evaluated $.2t^2$ at 3.1 (not 2.9)	6	0
Evaluated $.2t^2$ at 3.1 and 2.9 and subtracted or averaged	9	14
Error in evaluation of $.2t^2$	13	3
Error in differentiation	13	10
Other confused attempts	15	42
No attempt	9	15

Group B gave definite evidence of making a superior effort to find the correct solution to question 14, but yet even it did not make a good showing. There was much confusion--a great variety of inadequate responses.

Question 15.--Solve $dy/dx = x/2$.

TABLE XLV

ANALYSIS OF ERRORS MADE IN QUESTION 15

Explanation	Group B	Group C
Correct answer. $y = \frac{x^2}{2} + c$	32	7
Omitted 'c'	47	24
x^2	12	13
x^2 or $x^2 + c$	4	0
Other incorrect responses: $1/2x$, 1, x, etc.	11	31
No attempt	1	6

Members of group B were not vastly superior in their attempts to answer this question. Many in both groups forgot the constant of integration.

Question 16.--Evaluate $\int_{-1}^{1} 3x^2 dx$.

TABLE XLVI

ANALYSIS OF ERRORS IN D. I. QUESTION 1.

Explanation	Group 2	Group 3
Correct answer: 2	74	57
0 (Error in evaluating)	22	27
1d (differentiated instead of integrating: $6x$)	2	7
-1	0	4
Other incorrect answers: -2, 1, etc.	4	5
No attempt	0	2

Group 2 exceeded Group 3 in number of successes. Most of the errors were mechanical, and relating to signed numbers, e.g. $-(-2)$.

Question 17.--The first and second terms of an A.P. are a and b. What is the third term?

TABLE XLVII

ANALYSIS OF ERRORS MADE IN QUESTION 17

Explanation	Group B	Group C
Correct answer: $2b - a$	57	29
$a + 2b/a$	3	2
$a + b$	2	7
Incomplete, yet correct	5	3
$3a - 2b$, i.e. $a + 2(a-b)$	7	6
$a + b/a$, $b + b/a$, $b + a/b$	1	4
$a + 2d$	3	5
$a + 2b$	5	3
c	2	6
Other incorrect answers: ar^2, b, b^2/a, etc.	13	25
No attempt	2	7

The attainments of the students of the two groups were well distinguished here. Great was the variety of incorrect answers, indicating that only 62% of group B and 32% of group C knew how to form the terms of an arithmetic progression.

Question 18.--The second term of a G.P. is 10 and its common ratio is r. Find the first term.

TABLE XLVIII

ANALYSIS OF ERRORS MADE IN QUESTION 18

Explanation	Group B	Group C
Correct answer: 10/r	63	36
10r	4	5
10 - r (confused A.P. and G.P.)	2	17
1	4	5
r/10	2	2
10/a	3	1
Other incorrect answers: 5, ar, 11/r, etc.	17	18
No attempt	5	14

Attainments of the two groups were again well distinguished by the results of their efforts to solve question 18. Group B was definitely superior. Several group C students confused the arithmetic progression and the geometric progression.

Question 19.--Using pages 30-31 of Knott's tables, find the cube root of 200.

TABLE XLIX

ANALYSIS OF ERRORS MADE IN QUESTION 19

Explanation	Group B	Group C
Correct answer: 5.8480	55	41
5.85 (not accurate enough)	8	11
27.144 (used $10\sqrt{20}$)	7	9
2.7144 ($\sqrt[3]{20}$)	1	4
9.2832 ($2\sqrt[3]{100}$)	4	5
12.559 ($10\sqrt[3]{2}$)	1	5
8,000,000 (cube instead cube root)	2	3
14.142 ($10\sqrt{2}$)	3	2
4.6416 ($\sqrt[3]{100}$)	0	2
Other incorrect answers: 1.2559, etc.	9	6
No attempt	9	13

Both groups did as well as might be expected--group B being superior again. Many students did not see the relation between $\sqrt[3]{10n}$ and $\sqrt[3]{200}$, but tried complicated ways of solving in which they broke the rules of addition of surds and extractions of roots.

Question 20.--Using compound interest tables, find the value of $1/1.04^{12}$.

TABLE I

ANALYSIS OF ERRORS MADE IN QUESTION 20

Explanation	Group E	Group C
Correct answer: 0.62460	58	39
1/1.60103 (used 'amount' tables)	16	10
1.60103 ('amount' instead 'T.V.')	6	29
1/.6246 (used T.V. but put in denominator)	7	4
0.624, 0.62 (not accurate)	0	5
Other incorrect responses: 0.2460, 11.5340, etc.	2	7
No attempt	7	5

Group E students here once again were successful. Many students of both groups used a very roundabout way (using 'amount' tables and then dividing into 1) while many others merely gave the amount instead of the present value. This latter group could not have estimated their answers--otherwise they would have known that the answer had to be less than 1.

Question 21.--Evaluate ${}_6P_6$.

TABLE LI

ANALYSIS OF ERRORS MADE IN QUESTION 21

Explanation	Group B	Group C
Correct answer: 720	87	78
1 (${}_6C_6$)	10	13
0	0	3
Other incorrect	2	2
No attempt	1	4

Members of both groups did well with this question. Group B slightly excelled Group C. However, several students of both groups confused permutations and combinations.

Question 22.--In how many ways can these things be chosen from seven things?

TABLE LII

ANALYSIS OF ERRORS MADE IN QUESTION 22

Explanation	Group E	Group C
Correct answer: 35	59	34
210 ($^7P_3$)	30	21
840	1	7
Other errors in evaluating $^7C_3$	6	3
Incorrect in evaluating $^7P_3$	2	5
As far as $^7C_3$ only	3	1
As far as $^7P_3$ only	0	2
Other incorrect answers: $^7P_3$, $^7P_7$, $^{13}P_3$, etc.	0	6
No attempt	0	1

Members of group E were only slightly more successful than members of group C. Many students of both groups could not distinguish rules for evaluation of $^7C_3$ and $^7P_3$.

Question 23.—What is the first term in the expansion of $(2x^3 + y)^5$?

TABLE LIII

ANALYSIS OF ERRORS MADE IN QUESTION 23

Explanation	Group B	Group C
Correct answer: $32x^{15}$	35	27
Only as far as $(2x^3)^5$	8	11
$32x^8$ (incorrect evaluation)	10	9
$2x^{15}$	6	2
$2x^8$	6	10
$2x^3$	6	3
Other incorrect evaluations: e.g. $10x^5$	13	10
Other incorrect answers: e.g. y^5	12	22
No attempt	3	5

This question gave much difficulty to both groups. Most students knew how to form the first term of the expansion but could not apply the law of multiplication of indices. Once more members of group B were slightly more successful than members of group C.

Question 24.--Assuming that under certain conditions $(1 + x)^n \approx 1 + nx$, find an approximate value for $\sqrt[3]{1.06}$.

TABLE LIV

ANALYSIS OF ERRORS MADE IN QUESTION 24

Explanation	Group B	Group C
Correct answer: 1.02	78	50
1.18 (1 + 3x.06)	1	4
1.09 (1 + 3/2x.06	2	2
1.9 (error in evaluation)	0	2
1.353 (1 + 1/3x1.06)	0	3
Other incorrect answers: 1.0172, .36, .114, etc.	16	21
No attempt	2	17

Group B students received perfect scores in 28% more cases than did group C students. Several students of both groups were unable to apply the approximation formula.

Question 25.--Using the set of axes below, plot and label the points A(0.22, 10.8) and B(0.07, 13.5).

TABLE LV

ANALYSIS OF ERRORS MADE IN QUESTION 25

Explanation	Group E	Group C
Correct answer:	80	73
Plotted B abscissa as 0.007	9	4
Plotted A abscissa as 0.022	6	3
Plotted B ordinate as 10.35	3	3
Others: e.g. not labelled, etc.	7	15
No attempt	2	4

This problem in plotting presented little difficulty to either group. As usual, members of Group E were more successful.

Question 26.--Two numbers are in the ratio 3:7 and their sum is 9. What are the numbers?

TABLE LVI

ANALYSIS OF ERRORS MADE IN QUESTION 26

Explanation	Group B	Group C
Correct answer: 2.7, 6.3	55	40
Two equations but not solved	3	4
Confused with formula for sum of G.P.	2	4
Mechanical errors	3	3
Confused with formula for sum of A.P.	1	1
Other confused attempts	27	34
No attempt	12	15

Group B students again showed their superiority, yet many students in both groups made very confused attempts at a solution.

Question 27.--Find two numbers, x and y, such that $8 : x = x : y = y : 27$.

Table 2.11

ANALYSIS OF ANSWERS TO Q. 27

Explanation	Group E	Group G.
Correct answer: $x = 12$, $y = 18$	11	2
Gave two equations but are not solve or made error	29	24
Cross-multiplied or cancelled incorrectly	2	0
Confused concepts of ratio	14	19
Only 1 equation stated	9	12
Others: e.g. introduced constant 'k' but could go no further	21	15
No attempt	4	11

Answers of both groups indicated much difficulty with . Apparently they have come too much but to tackle a problem involving a continued proportion--the results showed ignorance or weakness. Again members of group E showed their superiority.

Question 28.--The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. If the resistance of an 8 foot wire having a diameter of 1/10 of an inch is 2 ohms, find the resistance of a 10 foot wire made of the same material and having a diameter of 1/16 of an inch.

TABLE LVIII

ANALYSIS OF ERRORS MADE IN QUESTION 28

Explanation	Group B	Group C
Correct answer: 5.4 ohms	58	43
Used correct statement of variation yet made error in finding value of k or r_2	23	28
$r \propto 1/\sqrt{d}$	1	4
Other incorrect answers	10	11
No attempt	1	1

Both groups were quite successful with this question--group B excelling again. Most errors were mechanical errors.

Question 29.--Sketch the graph of the function $\frac{3x - 2}{2x + 3}$ using the set of axes provided below.

TABLE LIX

ANALYSIS OF ERRORS MADE IN QUESTION 29

Explanation	Group B	Group C
Correct graph:	22	8
Horizontal asymptote missing or incorrect	64	80
Vertical asymptote missing or incorrect	41	61
x and y intercepts incorrect or not indicated	9	34
No attempt	2	2

Group B showed itself definitely superior again, yet even it met success in only 22% of its attempts. The students did not know how to discover the asymptotes.

Question 30.--If $y = 3(1.05)^x$, show that when $x = 2, 4, 6, 8$ the corresponding values of y are in G.P. and find the common ratio.

TABLE IX

ANALYSIS OF ERRORS MADE IN QUESTION 30

Explanation	Group B	Group C
Correct answer: $r = (1.05)^2$	13	4
Evaluated each y and found one ratio but was inaccurate	19	25
Evaluated y incorrectly or inaccurately	14	10
Mechanical error in evaluating ratios	9	19
Found r incorrectly (subtracted)	6	10
Correct answer but method not clear	10	5
Incomplete	9	20
No attempt	15	8

Group B students made more successful than students in group C, but all students found it difficult to give a complete solution.

Question 31.--AB is the diameter of a circle and P is any point on the circumference. If AB = 5 inches, (a) express the area of $\triangle APB$ (y square inches) as a function of AP (x inches), (b) give the range of x.

TABLE LXI

ANALYSIS OF ERRORS MADE IN QUESTION 31

Explanation	Group B	Group C
Correct answers:		
(a) $y = \frac{x}{2}\sqrt{25 - x^2}$	12	2
(b) $0 < x < 5$ or $0 \leq x \leq 5$	31	21
(a) Drew diagram only	21	26
$y = \frac{AP}{2}\sqrt{25 - AP^2}$	3	1
$y = 2.5\sqrt{x^2 - 25}$	2	1
$y = \frac{1}{2}(25 - x^2)$	7	4
Incomplete	14	27
Other confused attempts	32	29
No attempt	6	8
(b) Used symbols > and < inadvertently	3	5
0 to 5	5	2
0 to $2\frac{1}{2}$	3	2
$-5 \leq x \leq +5$	2	2
$-2.5 \leq x \leq +2.5$	1	1
Other confused attempts	7	13
No attempt	44	58

Students of group B lost again, yet the problem presented much difficulty. Their knowledge of analytical geometry is not adequate.

Problem 35.--Let $F(x)$ be a function defined on $[0,1]$ which satisfies all of the following conditions:

(1) $F(1) = 1$

(2) $F(x) \geq 0$ for $0 \leq x \leq 1$

(3) $F(x) + F(y) \leq F(x+y)$ for $0 \leq x \leq y \leq x+y \leq 1$.

Table Data

Verification of various values of Problem 35

Verification	Value 1	Value 2
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

Let $f(x)$ be a function defined for $0 \leq x \leq 1$ and satisfying the following conditions:

Question 17.--[illegible] = [illegible], find the average rate of change of y over the interval x = [illegible] to x = [illegible].

TABLE [illegible]

ANALYSIS OF ANSWERS TO QUESTION 17

Explanation	Group B	Group C
Correct answer: -1.2	14	7
Differentiated the function	11	[illegible]
[illegible]	4	14
[illegible]	6	[illegible]
[illegible]	1	6
[illegible]	11	12
[illegible]	7	5
[illegible]	[illegible]	4
[illegible]	12	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	17	21
No attempt	5	9

The students were not very successful in [illegible] either. They [illegible] of group [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] percent of group C.

Section 24.--

[illegible]

Table LXIV

[illegible]

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

Question 35.--If $r = s^2 + 2s$ and $s = t + 1/t$, express dr/dt as a function of t.

TABLE LXV

ANALYSIS OF ERRORS MADE IN QUESTION 35

Explanation	Group B	Group C
Correct answer: $dr/dt = 2t + 2 - 2/t^2 - 2/t^3$	28	9
Error in differentiation	23	31
Squared t + 1/t incorrectly	12	11
Confused dr/ds and dr/dt	6	14
Error in simplifying fractions	9	5
Other mechanical errors	10	2
Gave answer including both s and t	3	5
Incomplete solutions	4	2
Other incorrect solutions	14	30
No attempt	2	12

Once again members of group B showed themselves quite superior. Most marks were lost on account of breaking the rules of algebra and differentiation.

Question 36.--(a) Find the slope of the tangent to the curve $y = 3x^2 - 5x + 2$ at the point (2,4). (b) What is the slope of the curve at this point? (c) Find the equation of the tangent to the curve at this point.

TABLE LXVI

ANALYSIS OF ERRORS MADE IN QUESTION 36

Explanation	Group B	Group C
Correct answer: (a) 7	25	6
(b) 7	11	4
(c) $y = 7x - 10$	8	1
(a) $dy/dx = 6x-5$ (incomplete or incorrectly evaluated)	16	23
5/6	2	6
6	6	0
2 (i.e. 4/2)	12	10
Other incorrect answers	19	26
No attempt	22	29
(b) $6x - 5$	8	5
6	4	4
3	4	3
2	4	3
5/6	2	4
Other incorrect answers	14	23
No attempt	51	54
(c) $6x - 5$	7	2
Used incorrect value of x	15	11
Other incorrect answers	10	11
No attempt	60	75

Another question gave quite a lot of difficulty,--and as usual Group B was more successful than group C.

Question 37.--Expressing results without negative or fractional exponents, solve $dy/dx = 4x^2 - 3\sqrt{x} + 2/\sqrt{x}$.

TABLE LXVII

ANALYSIS OF ERRORS MADE IN QUESTION 37

Explanation	Group B	Group C
Correct answer: $4x^{3}/3 - 2\sqrt{x^3} + 4\sqrt{x} + c$	14	7
Omitted the constant c	63	75
Correct, except no c included	31	11
Differentiated	4	18
Second and/or third term(s) incorrect	47	70
No attempt	2	8

Again, all students experienced difficulty in dealing with surds, and group B was slightly superior to group C.

Question 7c.--A long rectangular strip of sheet iron 24 inches wide is made into an eavestrough with cross-section as shown in the diagram. The straight sides AB and EF are unequal, BC=EF and AB is one-half the radius of the semicircular base CDE. Find to the nearest tenth of an inch the width of the cross-section for which the trough will have greatest water-holding capacity. Show, at the proper place in your solution, that you have a "greatest" and not a "least".

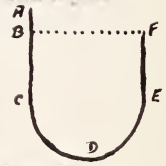

Table L.VIII

A 4-way of [illegible] in question 7c

Explanation	Group I	Group C
Correct answer: 11.3 inches	0	1
Mechanical error	4	6
Inconclusive	14	12
Omit and illegible	21	17
In attempt	41	37

This item very apparently was beyond the ability of both the groups of students. Very few attempted to provide and most of the attempts were extremely confused.

Question 10.--As shown in the accompanying diagram, a portion AB of the graph of the function $x^2 + 1$ has been rotated around the x-axis. Find the volume of the solid so generated, giving your answer to the nearest cubic meter.

TABLE XLII

ANALYSIS OF ANSWERS GIVEN TO QUESTION 10

Explanation	Group A	Group B
Correct answer: $\int_1^3 \pi(x^2+1)^2\,dx$	7	3
Incorrect limits used	55	35
Correct integration, incorrect limits used (or limits not given)	24	17
Omitted π	11	7
$\int_1^3 x^2+1\,dx$	13	13
Error in limits of integration	1	3
Correct integrand, left indefinite or integrated incorrectly	2	11
Others	0	1
No attempt	14	11

Discussion, also, was not difficult for these students (very groups).

Question 40.--(a) Find the "sum to infinity" of the following series: $1 - 1/2 + 1/4 - 1/8 + \ldots\ldots$

(b) if the series: $1 - r + r^2 - r^3 + r^4 \ldots\ldots$ has a "sum to infinity", find the range of r.

TABLE LXX

ANALYSIS OF ERRORS MADE IN QUESTION 40

Explanation		Group B	Group C
Correct answer:	(a) $S = 2/3$	49	23
	(b) $-1 < r < +1$	11	10
(a) 2 (used 1/1-r instead of 1/1+r)		11	9
Used general formula for sum of G.P.		6	10
Mechanical error in evaluation		2	6
1		2	3
Used incorrect formula		4	1
-1		1	2
$\frac{1}{2}$		2	1
Confused with A.P.		2	1
-2		2	1
Incomplete		2	3
Confused attempt		7	12
No attempt		12	30
(b) $0 < r < 1$		4	2
Gave no range--did not understand the term 'range' but described it (i.e. 'sum'		19	19
Confused the symbols > and <		8	4
$-\infty$ to $+\infty$		3	4
Used symbol '∞' incorrectly--often as denominator of fraction		2	2
Other confused attempts		13	6
No attempt		30	51

Group E students did twice as well as Group U students on this item. Students could have earned three 'easy' marks in Part (b) had they known the meaning of 'range of a variable'.

Question 41.--On December 1, 1937 a man invested $500. He and continued to invest this same amount on each succeeding December 1. If these deposits earn interest at the rate of 4% compounded annually, find the total amount to his credit immediately after he has made his deposit on December 1, 1944.

TABLE XXXI

ANALYSIS OF ERRORS MADE IN QUESTION 41

Explanation	Group B	Group C
Correct series $500 + $507.76) ..... to three. Or : $8,.07.	4	5
8 terms only ($7 or less)	21	21
Used present value--series 9 1 term only: e.g. $507(1.04)^{9} = $711.15	15	17
Series of records correct but not by the tables	17	11
Used first term as $500(1.04)	9	8
Other confused ideas	3	4
Has terms--e.g. $500 + $500 (1.04)^{2}	17	7
No attempt	5	7
	10	6

There is a wide lack of such confusion on the parts of members of group B and members of group C. Students could not distinguish 'amount' from 'present value', nor did they realize, in many cases, that ten deposits were made in all.

Question #2.--Ten members of one club and twelve members of another want to choose a committee of three. If both clubs must be represented on the committee, how many different committees are possible.

Table xxxii

Analysis of Errors in Items 1 and 2, Question #2

Errors	Group 1	Group 2
Correct answer: $\frac{12!}{3!9!} \times \frac{10!}{2!8!}$	11	12
Wrong formula: $\frac{12!}{3!9!} \times \frac{10!}{2!8!}$	0	1
$\frac{12!}{3!9!} + \frac{10!}{2!8!}$	0	11
Mathematical error	1	2
Used permutations instead of combinations	1	11
$\frac{12!}{3!9!} \times \frac{10!}{2!8!} \times \frac{22!}{3!19!}$	7	1
$\frac{12!}{3!9!} \times \frac{10!}{2!8!}$	1	1
$\frac{12!}{3!9!} \times \frac{10!}{2!8!} \times \frac{12!}{3!9!} \times \frac{10!}{2!8!}$	1	1
$\frac{12!}{3!9!} \times \frac{10!}{2!8!}$	2	1
Other (Wrong mathematical answer)	2	1
Incomplete	4	2
Correct alternate	17	23
No answer	7	6

Students of group 2 handled this problem better than students of group 1, but neither group did well. 71% of the students in group 1 and 92% of the students in group 2 had a knowledge of all that the problem--did not know how to tackle it or

even how to estimate an answer for their responses varied from '2' to '42,768,300'.

Question 43.--Write down and simplify the first three and the last three terms in the expansion of $(x^2 - 1/x)^{10}$.

TABLE LXXIII

ANALYSIS OF ERRORS MADE IN QUESTION 43

Explanation	Group B	Group C
Correct answer: $x^{20} - 10x^{17} + 45x^{14} \ldots 45/x^4 - 10/x^7 + 1/x^{10}$.	9	1
First terms given as x^{12} or other incorrect expression	24	30
Terms not simplified	26	17
Incorrect sign(s)	16	18
Numerical coefficients incorrect	17	15
Incorrect set-up for terms of the series	15	26
Other errors in indices	4	0
Other algebraical errors	4	1
Factors made separate terms	1	2
Incomplete	5	11
No attempt	13	20

Nearly all students experienced difficulty with question 43, with just a few more perfect scores for group B than for group C. It appears that the students showed weakness when forced to deal with indices and the binomial theorem.

Question 44.--The following table contains a few of the results obtained in an experiment.

P	3.30	4.34	5.70	7.50	10.0
h	6	7	8	9	10

Plot P/h against P and find the best-fit equation expressing P in terms of L. (Use the squared paper on page 13.)

TABLE LXXIV

ANALYSIS OF PAPERS FOR QUESTION 44

Explanation	Group B	Group C
Correct answer: $P = 5h/(13-h)$	0	0
Plotted P against P/h	39	30
Plotted P against h or h vs.P	18	22
Just not accurate enough	17	9
Plotted correctly, but used h and c (or a and c only) in equation	21	13
No equation given	12	25
Used poor scale	8	11
Mechanical error	0	1
Incomplete	1	1
Other somewhat correct	1	4
No attempt	7	4

This question asking the students to form the equation of a function pictured in a graph, gave great difficulty. Many students did not follow the instructions exactly. Others used poor scales for plotting or plotted inaccurately.

Conclusion

Group D students consistently showed superiority over group C students. Sometimes this superiority was very slight, while at other times it was great. It may be concluded, then, that these students in group D merited the higher grading they received on the Algebra 2 final examination. However it must also be admitted that even the group D students did not have an acceptable background for university mathematics and science. They showed weakness in carrying out elementary mathematical operations and in the application of the principles of analytical geometry to simple problems.

CHAPTER IV

VALIDITY OF THE VARIOUS TEST ITEMS

Relative Difficulty of the Items

The success of the students with the different items varied greatly. Some questions both groups found extremely easy: others both groups found too difficult. The remaining questions are the ones that really distinguish the two groups.

The following table indicates the questions for which more than 70% of the students of either group received perfect scores.

TABLE LXXV

THE EASIER ITEMS

Question	Topic	Group(s) Making Perfect Scores in 70% of the cases (or more)
1	Fractions in arithmetic	B and C
2	Proportion	B and C
3	Inverse variation	B
4	Solution of cubic function	B
7	Solution of a quadratic	B
8	Evaluation of a function	B and C
12	Differentiation of simple function	B and C
16	Evaluation of a definite integral	B
21	Permutations	B and C
24	Approximation (binomial theorem)	B and C
25	Plotting	B and C

At the other extreme are the questions found too difficult. These are outlined in Table LXXVI.

TABLE LXXVI

THE MORE DIFFICULT ITEMS

Question	Topic	Group(s) Making Perfect Scores in less than 10% of the Cases
9	Solution of quadratic using formula	C
11	Finding gradient, given intercepts	C
14	Approximate change in a function	C
15	Integration of simple fcn.	C
27	Continued proportion	B and C
29	Sketching a function	C
30	Finding a common ratio of GP.	B and C
31	Analytical geometry	B and C
32	Finding quadratic satisfying given conditions	B and C
33	Finding average rate of change	B and C
35	Derivative of a dependent variable	C
36	Slope of tangent and curve	B and C
37	Differentiation (negative and fractional exponents)	B and C
38	Maxima capacity	B and C
39	Volume of generated solid	B and C
40	'Sum to infinity', recog.	B and C
41	Annuity (amount)	B and C
42	Combinations	C
43	Binomial theorem	B and C
44	Empirical formula	B and C

There were, apparently, more relatively difficult questions than relatively easy ones. For questions 38 and 44 not a single perfect score was awarded in these two groups consisting of 200 stu-

dents. These questions were likely too difficult.

However, one must at the same time keep in mind the time element. Quite likely the examination paper was designed in such a way that no student would be expected to complete the paper. It also seems reasonable to suppose that the first items would be handled first, thus giving more perfect scores on the above answer problems at the beginning of the test.

Items 2, 5 and 12 were found extremely easy by both groups. Over 10% of the students received perfect scores and the difference in percent success of the two groups was only 6. These items, too, then, did not discriminate the abilities of the two groups very well.

Items for which the difference in percent success was at least 15 are given in Table LXVII.

TABLE LXXVII

GOOD ITEMS

Question	Topic	Difference in % Success
3	Inverse variation	16
4	Solution of cubic function	17
5	Forming a function	16
6	Using graph to solve equation	22
7	Finding C in given equation	19
9	Solving equation using formula	17
14	Approximate change in function	16
15	Integration of simple expression	15
16	Evaluating a definite integral	15
17	Finding third term of A.P.	28
18	Finding first term of G.P.	27
20	Using compound interest tables	19
24	Approximation (binomial)	28
26	Formula, solving two equations	15
28	Problem (variation)	15
34	Differentiation (fractional and negative exponents)	31
35	Derivative of dependent variable	19

From the point of view of distinguishing between the attainments of Groups B and C, these seventeen items were very good. However, questions 9, 14, 15 and 35 were too difficult: questions 3, 4, 6, 7, 16 and 24 were too easy. If these are eliminated, the best items would then be: 5, 17, 18, 20, 26, 28 and 34.

The Test as a Whole

The highest possible total raw score was 197. The highest mark given anyone was 152. This was called 100%. The lowest mark given anyone was 0 and this was called 0%. If members of group E had been successful in obtaining 50% of the total possible raw score they would have obtained at least 78, whereas actually they scored only 65 (lower limit). Similarly, if members of group C had been successful in obtaining 40% of the total possible raw score they would have obtained at least 63, whereas actually they scored only 42 at the lower limit.

The question could be raised as to whether or not students might find this situation rather discouraging.

Another way of attempting to discover the validity of the various test items might be to compile statistics showing how many students in group E were given approximately 50% (or more) of the marks for a given question and how many students in group C were given approximately 40% (or more) of the marks allotted for it.

The next table shows the frequency with which group B members received 40% or more of the marks allotted for a given question (questions 26 to 44 only).

TABLE LXXVIII

FREQUENCIES OF 40% OR MORE OF MARKS ALLOTTED GROUP B

Question	Topic	Possible Score	Score	Freq.
26	Forming, solving eqns.	4	2 or more	57
27	Continued proportion	6	3 or more	11
28	Problem in variation	6	3 or more	72
29	Sketching a function	6	3 or more	57
30	Common ratio or G.P.	4	2 or more	20
31	Analytical geometry	6	3 or more	23
32	Forming function	6	3 or more	30
33	Av. rate of change	5	2 or more	34
34	Differentiation	5	2 or more	83
35	Differentiation	6	3 or more	52
36	Slopes of tangent, curve	5	2 or more	16
37	Integration	6	3 or more	75
38	Maximum value	7	3 or more	0
39	Integration (volume)	6	3 or more	35
40	Sum to infinity, range	6	3 or more	31
41	Annuity (amount)	6	3 or more	16
42	Combinations	6	3 or more	29
43	Binomial theorem	5	2 or more	32
44	Empirical formula	6	3 or more	31

According to this test the best items were 28, 34 and 37. Table LXIX shows a similar survey for group C.

TABLE LXXIX

FREQUENCIES OF 30% OR MORE OF MARKS ALLOTTED
GROUP C

Question	Topic	Possible Score	Score	Freq.
26	Forming, solving eqns.	4	2 or more	44
27	Continued proportion	6	2 or more	20
28	Problem in variation	6	2 or more	77
29	Sketching a function	6	2 or more	53
30	Common ratio of G.P.	4	2 or more	15
31	Analytical geometry	6	2 or more	10
32	Forming function	6	2 or more	9
33	Av. rate of change	5	2 or more	17
34	Differentiation	5	2 or more	56
35	Differentiation	6	2 or more	39
36	Slopes of tangent, curve	5	2 or more	9
37	Integration	6	2 or more	36
38	Maximum value	7	3 or more	0
39	Integration (values)	6	2 or more	25
40	Sum to infinity, range	6	2 or more	31
41	Annuity (amount)	6	2 or more	10
42	Combinations	6	2 or more	16
43	Binomial theorem	6	2 or more	13
44	Simplified formula	6	2 or more	20

Item 28 ranks quite high again. Considered from three different viewpoints, then, item 28 is found to be the most valid test item. Item 34 is satisfactory from two of the viewpoints.

Conclusion

The test items differed greatly in difficulty. Some were found extremely easy by both groups. Others were found too difficult by both groups. Still others were found to differentiate between the two groups very well. Those that distinguished the attainments of the two groups well were: 5, 17, 18, 20, 26, 28, 34 and 37, with special mention of items 28 and 34.

CHAPTER V

GENERAL SUMMARY AND CONCLUSIONS

Perhaps the examination was a little too difficult. Taken as a whole, however, it was a valid test in the sense that it separated students into two groups consistently--that is, group B students excelled group C students in their efforts to answer 43 out of the 44 questions.

This survey indicates a weak mathematical background for most students. Perhaps teachers of mathematics might take time to stress more of the fundamentals. Perhaps, on the other hand, some of the more sophisticated exercises in the textbook might be replaced with drills on the simpler applications. Or again, perhaps the course outlined is too vast to be humanly possible in the allotted time, especially during the 'war years'.

The finding of so many mechanical errors would also indicate a need for insisting upon accuracy of details and for more drill upon fundamentals during earlier school years.

It is hoped that this report will give guidance to instructors of mathematics in that it

will show where the weaknesses tend to occur and thus aid in remedial work.

It is hoped, too, that this report will be of some value in the planning of future courses outlined and final examination papers for grade twelve algebra.

Another point of interest might be noted here. The above table questions 1 to 17 distinguish the weaker Matric group B from the other just as surely as do the longer questions. Tables XXVII and XXVIII demonstrate this very clearly. However, it must be admitted that for remedial purposes the longer problems are of more value for they show more exactly what error is made.

Table XXXX shows how group B excelled group C. As stated previously, group B students received more correct answers than did group C students. Group C students left more items incomplete. These weaker students also omitted many more questions, quite likely because they lacked insight into these problems. Group C students made more errors in interpretation and integration. They also misinterpreted more questions. In many more

cause they confused the arithmetic progression with the geometric progression, combinations with permutations, and methods of finding the amount and the present value of an annuity. Group C students also had more difficulty with the binomial theorem and with proof work.

The fact that question 1 was answered correctly by 91 Group C students and only 75 Group E students may be attributed to chance. For two populations of 100 individuals each a difference of 4 is not significant. Moreover the students with raw scores of 40 and of 71 were chosen at random.

Order of questions. Figures 7 and 8 indicate that, on the whole, items 1 to 25 were easier than items 26 to 44. For items 26 to 44 the general trend is toward greater difficulty. The easier and the more difficult items appear to be interspersed--no questions 1 to 25. Neither trend is regular. Figure 7 has a tendency to be U-shaped, especially if items 8, 12 and 16 were neglected. This would indicate that the easier questions were numbered 1 to 4 and 22 to 25.

However the graph is so irregular that the first suggestion that the most and little-it items were interchanged seems the better.

TABLE LXXX

HOW GROUP B EXCELLED GROUP C
SUMMARY OF ANALYSIS OF ERRORS[a]
(QUESTIONS 1 TO 44)

Explanation	Group B	Group C
Correct answer	2011	1427
No attempt	530	770
Mechanical errors	206	178
Mechanical errors as percentages of numbers of attempts	33.5%	46.8%
Incomplete questions	148	193
Errors in differentiation	43	61
Errors in integration	61	80
Misinterpreted question (e.g. differentiated instead of integrating)	40	64
Gave answer in terms of variable (e.g. $\bar{x}$) instead of concrete number or used incorrect variable (e.g. x,y instead $\bar{x}$,h)	57	57
Language difficulty, e.g. meaning or determination of intercepts or range	96	92
Confused I.P. and S.P.	13	33
Confused combinations and permutations	48	60
Confused 'amount' and 'present value'	37	57
Inaccurate plotting or use of poor scale	192	241
Difficulty with binomial theorem	84	111
Misused symbols: $=, \neq, \infty, \propto, >, <, \int$, etc.	64	69

[a] Separate parts of questions were treated as separate questions.

WHAT A TYPICAL GROUP 5 STUDENT KNOWS:

1. How to find a fraction with a given denominator, equivalent to a certain fraction.
2. How to find the second term in a proportion.
3. How to find a variation-relation.
4. How to express the area of a square as a function of its perimeter.
5. How to solve an equation, given the graph of the function.
6. How to find the constant 'c' in an equation, given that the roots are equal.
7. How to find the value of $f(-1)$ for a given function.
8. How to find the limit of a function as the variable increases without limit.
9. How to find the gradient of a straight line, given its x and y intercepts.
10. How to differentiate a simple function.
11. How to find the maximal value of a given function.
12. How to evaluate a definite integral.
13. How to find the third term of an A.P. given the first and second terms.
14. How to find the first term of a G.P., given the second term and the common ratio.
15. How to find the gradient of $\sin^2 x$ using Leibnitz's Theorem.
16. How to evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x \, dx$.
17. How to find the number of ways of choosing three things from seven.
18. How to find the approximate value for the cube root of 1.06.
19. How to find the median.
20. How to find the average rate of change of a given function over a given interval.
21. How to differentiate a given function, expressing results without negative or fractional exponents.
22. How to find the sum of r in the series $1 - r + r^2 - r^3 + \ldots\ldots$ to n terms to infinity.

What a typical math I student NEED TO KNOW:

1. How to find at what three points a cubic function crosses the x-axis.
2. How to solve an equation using the formula.
3. How to find the x-coordinate change in a given function.
4. What a constant of integration is required.
5. How to evaluate $\int_1^{12}$ using computer instead of tables.
6. How to find the first term in the expansion of $(2x^2 + y)^7$.
7. How to find the geometric, given their ratio and sum.
8. How to find the second and third terms of a continued proportion.
9. How to solve a problem in variation.
10. How to locate asymptotes of a function.
11. How to show that the graph of a cubic has a d.p. and how to find the coordinates.
12. How to find an algebraic expression for a function and how to find the range of the variable.
13. How to form a quadratic equation with roots whose reciprocals etc.
14. How to square $(b + 1/b)$.
15. How to find the slope of the tangent to a curve, the slope of the curve and the equation of the tangent to the curve, at a given point.
16. How to substitute a formula expressing the result (given) in terms of practical variables.
17. How to solve a difficult problem involving a maximum value.
18. How to find the values of a scale of graduation.
19. How to find the 'mean to fluctuate' of a given curve.
20. How to find the amount of an annuity.
21. How to solve a problem dealing with 'combinations'.
22. How to solve a problem dealing with the 'binomial expansion'--writing of terms.
23. How to find a best-fit equation for a given set of data.

WHAT A TYPICAL GROUP OF STUDENT KNOWS:

1. How to find a function with a given derivative, equivalent to a certain problem.
2. How to find the general term in a progression.
3. How to find the lowest limit at which a cubic function crosses the x-axis.
4. How to solve an equation, given the graph of the function.
5. How to find the constant 'k' in an equation, given that the roots are equal.
6. How to find the value of $f(-1)$, for a given function.
7. How to differentiate a simple function.
8. How to find the third term of an A.P. given the first and second terms.
9. How to evaluate $1.1.04^{12}$ using compound interest tables.
10. How to evaluate $\sum_{1}^{6}$.
11. How to find the number of ways of choosing three things from seven.
12. How to find an approximate value for the gradient of 1.1^{x}.
13. How to find the numbers, from basic ratio and sum.
14. How to solve a problem in statistics.
15. How to find the term to infinity, of a given series.

What a TYPICAL SHEET of a "Round Robin" set is like:

1. How to find a variation-relation.
2. How to express the area of a square as a function of its perimeter.
3. How to solve an equation using the formula.
4. How to find the limit of a function as the variable increases without limit.
5. How to find the gradient of a straight line, given its x and y intercepts.
6. How to find the smallest value of a given function.
7. How to find the approximate change in a given function.
8. How to integrate a simple harmonic motion.
9. How to evaluate a definite integral.
10. How to find the first term of a G.P., given the second term and the common ratio.
11. How to find the nth root of N using Newton's method.
12. How to find the first term in the expansion of $(ax^2 + y)^5$.
13. How to plot a point.
14. How to find the period and the range of a sine-wave projection.
15. How to locate a horizontal asymptote.
16. How to find the area of a sector of an x.y. and how to use the compass.
17. How to find graphically an approximation of a function and how to find the roots of the function.
18. How to form a quadratic function which meets three conditions.
19. How to find the average rate of change of a given function over a given interval.
20. How to differentiate a given function, expressing the results without negative or fractional exponents.
21. How to find dr/dt given r as a function of s and s as a function of t.
22. How to find the slope of the tangent to a curve, the slope of the curve and the equation of the tangent to the curve, at a given point.
23. How to integrate a function expressing the results without negative or fractional exponents.

24. How to solve a difficult problem involving a maximum value.
25. How to find the volume of a solid of revolution.
26. How to find the range of r in the series $1 - r + r^2 - r^3 + \ldots\ldots$ has a 'sum to infinity'.
27. How to find the amount of an annuity.
28. How to solve a problem dealing with combinations.
29. How to find the terms of a binomial expansion.
30. How to find a best-fit equation for a given set of data.

WHAT A FRIEND KNOWS ABOUT A STUDENT'S MIND.

(second sample)

1. How to find a fraction with a given denominator equivalent to a certain fraction.
2. How to find a variation-relation.
3. How to find the three points at which a cubic function crosses the x-axis.
4. How to find the constant 'c' in an equation, given that the roots are equal.
5. How to find the value of f(-1) for a given function.
6. How to find the limit of a function as the variable increases without limit.
7. How to differentiate a simple function.
8. How to find the smallest value of a given function.
9. How to evaluate a definite integral.
10. How to find the third term of an A.P. given the first and second terms.
11. How to find the value risk of 200 using annuity tables.
12. How to evaluate $1/1.04^{12}$ using compound interest tables.
13. How to evaluate $^{n}C_{r}$.
14. How to find the number of ways three things can be chosen from seven.
15. How to find an approximate value for the cube root of 1.06.
16. How to plot the results.
17. How to find two numbers given their ratio and sum.
18. How to find the second and third terms of a continued proportion.
19. How to solve a problem in variation.
20. How to show that the terms of a series are in G.P. and how to find the common ratio.
21. How to find the sum to infinity of a series.

WHAT A TYPICAL GRADE 12 STUDENT KNOWS AND CAN DO:

(second sample)

1. How to find the second term of a progression.
2. How to express the area (A) of a square as a function of its perimeter.
3. How to solve an equation given the graph of the function.
4. How to solve an equation using the formula.
5. How to find the gradient of a straight line given the x and y intercepts.
6. How to find the numerical rate change in a functio
7. That a constant of integration is required when integrating.
8. How to find the first term of a G.P. given the second term and the common ratio.
9. How to find the first term in the expansion of $(2x^2 + y)^5$.
10. How to locate asymptotes.
11. How to find analytically an expression for a function and how to find the range of the variable.
12. How to form a quadratic function which meets three requirements.
13. How to find the average rate of change of a given function over a given range.
14. How to differentiate a given function, expressing results without negative or fractional exponents.
15. How to square $(x + 1/x)$.
16. How to find the slope of the tangent to a curve, the slope of the curve and the equation of the tangent, at a given point.
17. How to integrate a function expressing the result without negative or fractional exponents.
18. How to solve a difficult problem involving a maximum value.
19. That, when integrating to find the volume of a solid of revolution, the formula includes a π.
20. How to find the range of the common ratio of the terms of a G.P. when the series has a sum to infinity.
21. How to find the amount of an annuity.
22. How to solve a problem dealing with combinations.
23. How to find the terms in a binomial expansion.
24. How to find the best-fit equation for a set of data.

WHAT A TYPICAL GOOD C STUDENT KNOWS:

(second sample)

1. How to find the second term of a proportion.
2. How to find a variable-relation.
3. How to find the three points at which the graph of a cubic function crosses the x-axis.
4. How to find the value of the constant 'C' in an equation, given that the roots are equal.
5. How to find the value of f(-1) for a given function.
6. How to find the limit of a function as the variable increases without limit.
7. How to differentiate a simple function.
8. How to find the smallest value of a given function.
9. How to evaluate a definite integral.
10. How to find the third term of an A.P. given the first and second terms.
11. How to find the number of ways of choosing three things from seven.
12. How to find two numbers, given their ratio and sum.
13. How to solve a problem in variation.
14. How to find analytically an expression for a function.
15. How to find $\frac{dy}{dx}$ given x as a function of s and s as a function of t.

WHAT A TYPICAL GOOD - STUDENT DOES NOT KNOW:

(second sample)

1. How to find a function with a given derivative, equivalent to a certain function.
2. How to represent the area of a square as a function of its diagonal.
3. How to solve an equation given the graph of the function.
4. How to solve an equation using the formula.
5. How to find the gradient of a straight line, given its x and y intercepts.
6. How to find the x and y co-ords of any point on a function.
7. How to integrate a simple function.
8. How to find the sum of a G.P. given the first term and the common ratio.
9. How to find the mean mark of 200 boys, Cathy's inclusive.
10. How to evaluate $1,1.06^{12}$ using compound interest tables.
11. How to evaluate $^{n}P_{r}$.
12. How to find the first term in the expansion $(ax^2 + y)^5$.
13. How to find an approximate value for the cube root of 2.06.
14. How to plot two points.
15. How to give the general principle right at a particular question.
16. How to square it.
17. How to locate two points.
18. How to know that the area under the curve $y = \frac{1}{x}$ is $\ln x$ and how to find the second term.
19. That $\sqrt{2} - x^2$ does not equal $2 - x^2$, and how to find the range of x.
20. How to form a quadratic function which passes through two points.
21. How to find the average rate of change of a given function over a given interval.
22. How to differentiate a given function, expressing results without negative or fractional powers.
23. How to find the slope of the tangent to a curve, the slope of the curve and the equation of its tangent, at a given point.
24. How to integrate a function, by giving the results without negative or fractional powers.

25. How to solve a difficult problem involving a maximum value of a function.
26. How to find the volume of a solid of revolution.
27. How to find the 'sum to infinity' of a given series and how to find the range of r in the series $1 - r + r^2 - r^3 + \ldots\ldots$ if this series has a 'sum to infinity'.
28. How to find the amount of an annuity.
29. How to solve a problem dealing with combinations.
30. How to find the terms of a binomial expansion.
31. How to find a best-fit equation for a given set of data.

The samples chosen from Group B are not identical. The thought of chance has entered. No two individuals are exactly alike--one would expect that different individuals, even from the same group, might be unlike in their success with at least a few of the items. Similarly, and for the same reason, the samples from Group C are not identical.

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